AST 1420

Galactic Structure and Dynamics

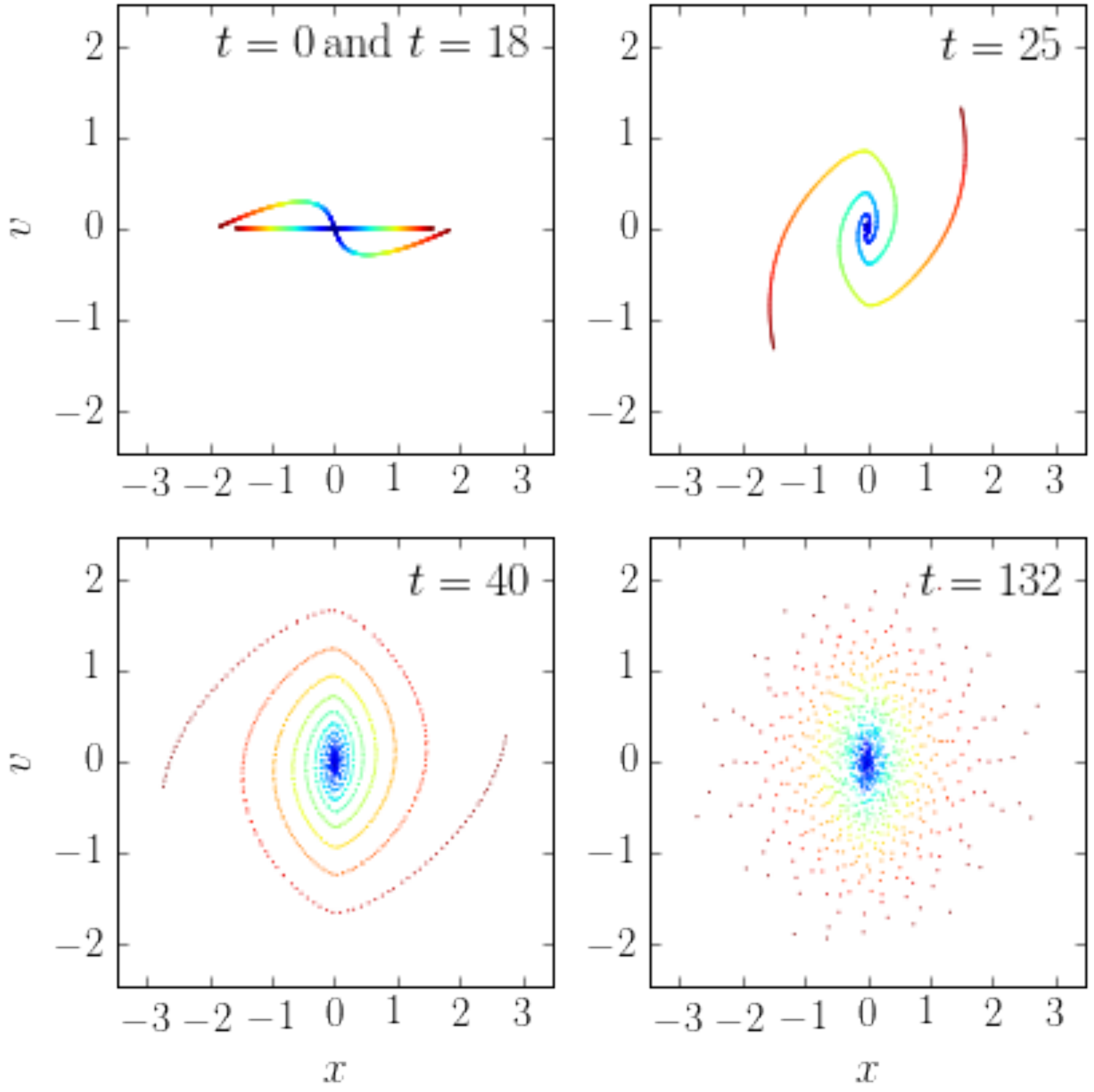
Q&A

Assignment 1 posted on the website!

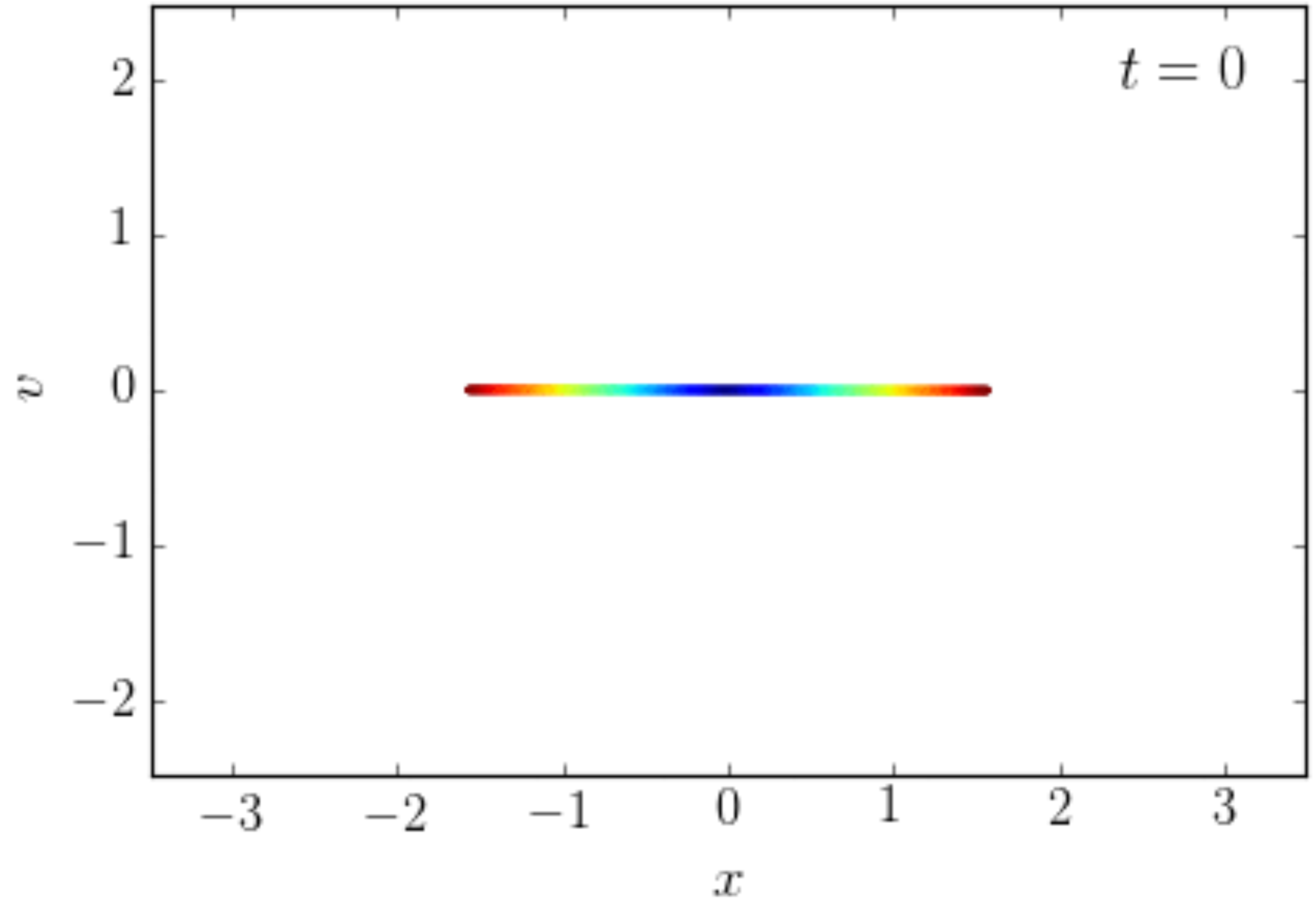
Questions submitted

- Violent relaxation and phase mixing
- Relaxation time
- Virial theorem and GC application
- Collisionless Boltzmann equation
- Jeans equations
- Jeans theorem and distribution functions
- Applications:
    - Dwarf spheroidals
    - Local Group timing

Violent relaxation and phase mixing



t = 0 and t = 18
t = 25
t = 40
t = 132
v
x



t = 0
v
x

Relaxation time

Relaxation time and collisional vs. collisionless systems

- Why are we able to ignore the dynamics of individual stars on each other? What about binaries, triples, etc.?
    - Basically, the relaxation-time argument is that the density is so low in galaxies that close encounters are very rare
    - Of course, this ignores binaries, stars born in clusters, and other stellar associations and only holds for the dynamics of 'field stars'
- Use of the impulse approximation and ignoring very strong encounters:
    - For ~constant density medium, expected velocity-squared changes scale as db/b (b: impact parameter), so strong encounters only contribute a very small part of the total expected velocity change.
    - This will be relevant when we discuss N-body simulations!
- Dense systems will through collisions evolve to a thermal state, similar to gas; galaxies do not

The viral theorem

The virial quantity

- $G = \text{sum}_i\ w_i\ \mathbf{x}_i\ .\ \mathbf{v}_i$
- For wi = mi, G is twice the time derivative of the scalar moment of inertia (we'll use that later to derive the tensor version of the virial theorem)
- That G is conserved is an *assumption* about equilibrium
- If you don't set wi = mi, need to be careful to not break the assumption of equilibrium (so you can't be too specific about particle identity)
- wi =/= mi is used to create different *mass estimators*, but in practice need to test these with simulations

Milky Way mass from globular cluster kinematics

- Why does the self-gravitating case work so well?
  - Estimator is still a ratio of squared velocities to inverse positions, so as long as you combine these in a sensible way, won’t be that far off!

Collisionless Boltzmann equation

Phase space density

- What is f(**x**,**v**) = f(**w**)?
  - Probability of seeing a body in volume d**x** d**v**, similar to spatial number density
  - If defined such that \int d**x** d**v** f(**x**,**v**) = N, then it's the number of bodies in a small volume
  - *Always* a probability of seeing a body in this 6D volume, even when written as f(E) or f(E,L)

Jeans equations

Just a moment…

- What's a moment?
- Concept in probability theory, for probability density p(x), moment is any function

  \int dx x^a p(x)

  [note that \int dx p(x) = 1]

- For the Jeans equations, we consider moments of

  p(**v**) = f(**x**,**v**)/ \int d**v** f(**x**,**v**) = f(**x**,**v**) / nu(**x**)

  [where nu(x) is the number density]

$$\nu(\mathbf{x}) = \int \mathrm{d}\mathbf{v}\, f(\mathbf{x}, \mathbf{v}) \,,$$

$$\bar{\mathbf{v}}(\mathbf{x}) = \frac{1}{\nu(\mathbf{x})} \int \mathrm{d}\mathbf{v}\, \mathbf{v} f(\mathbf{x}, \mathbf{v}) \,,$$

Integrating the CBE

$$\frac{\partial f(\mathbf{q},\mathbf{p},t)}{\partial t} + \dot{\mathbf{q}}\,\frac{\partial f(\mathbf{q},\mathbf{p},t)}{\partial \mathbf{q}} + \dot{\mathbf{p}}\,\frac{\partial f(\mathbf{q},\mathbf{p},t)}{\partial \mathbf{p}} = 0\,,$$

- **qdot** and **pdot** are given by Hamilton's equations, for Cartesian coordinates this means that

$$\frac{\partial f(\mathbf{x},\mathbf{v},t)}{\partial t} + \dot{\mathbf{x}}\,\frac{\partial f(\mathbf{x},\mathbf{v},t)}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{x}}\,\frac{\partial f(\mathbf{x},\mathbf{v},t)}{\partial \mathbf{v}} = 0\,,$$

- Integrating then gives

$$\int \mathrm{d}^3\mathbf{v}\,\frac{\partial f}{\partial t} + \int \mathrm{d}^3\mathbf{v}\,v_i\frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i}\int \mathrm{d}^3\mathbf{v}\,\frac{\partial f}{\partial v_i} = 0,$$

Spherical Jeans equation and anisotropy

$$\frac{\mathrm{d}(\nu \overline{v_r^2})}{\mathrm{d}r} + \nu \left(\frac{\mathrm{d}\Phi}{\mathrm{d}r} + \frac{2\overline{v_r^2} - \overline{v_\theta^2} - \overline{v_\phi^2}}{r} \right) = 0 \, .$$

- Assuming zero rotation gives

$$\frac{\mathrm{d}(\nu \overline{v_r^2})}{\mathrm{d}r} + 2\frac{\beta}{r} \nu \overline{v_r^2} = -\nu \frac{\mathrm{d}\Phi}{\mathrm{d}r} \, .$$

- What if rotation is non-zero?
    - Assume some rotation and plug it in!
    - Dark matter halos are not believed to have much rotation, neither does, e.g., the globular-cluster system

Jeans theorem and distribution functions

Integrals of the motion

- Integrals of the motion *I* are quantities that depend on (**x**,**v**) and are conserved along the orbit
- They cannot explicitly depend on time
- Examples: energy in a static potential, angular momentum in a spherical potential
- *Constants of the motion* do depend on time and are not terribly useful
- Existence of integrals of the motion typically limits the orbit to a lower-dimensional subspace (e.g., energy, angular momentum). Those integrals are called *isolating*

The King profile

- Remember: King profile is a way to deal with the infinite nature of the isothermal sphere

- Do this by introducing a cut off in the distribution function

$$f(\mathcal{E}) = \frac{\rho_1}{(2\pi\,\sigma^2)^{3/2}}\left[\exp\left(\frac{\Psi - \frac{v^2}{2}}{\sigma^2}\right) - 1\right]$$

- Where does this come from? Is there a good reason for this type of cut off?
    - Basically, it's one of the simplest ways to cut off the distribution function
    - Happens to fit elliptical galaxies and globular clusters well
    - There are other ways to cut it off that are less harsh

Applications to masses of spherical systems

Dwarf spheroidals

- Why do they have so much dark matter?
- Question for galaxy formation! Multiple reasons:
    - Gas cooling in low-mass dark-matter halos is not efficient at all but the earliest times
    - Lowest mass dwarf spheroidals: gas is removed during reionization
    - Once galaxies fall into another galaxy, they cannot accrete more gas and thus their fuel for star formation is cut off
- All work together to increase the ratio of dark to baryonic matter in low mass galaxies

Local group timing

- Why do the Milky Way and M31 have to be in their first radial period?
- Because of their radial orbit, they pass through each other at the end of the first radial period and that would lead to a merger that the galaxies would not survive