# AST 1420

# Galactic Structure and Dynamics

# Reminders

- Final posted (see slack), due Dec. 17
- Please sign up for an exam slot on Dec. 18 (see slack for Doodle link)
- Course evaluations *due tonight!*

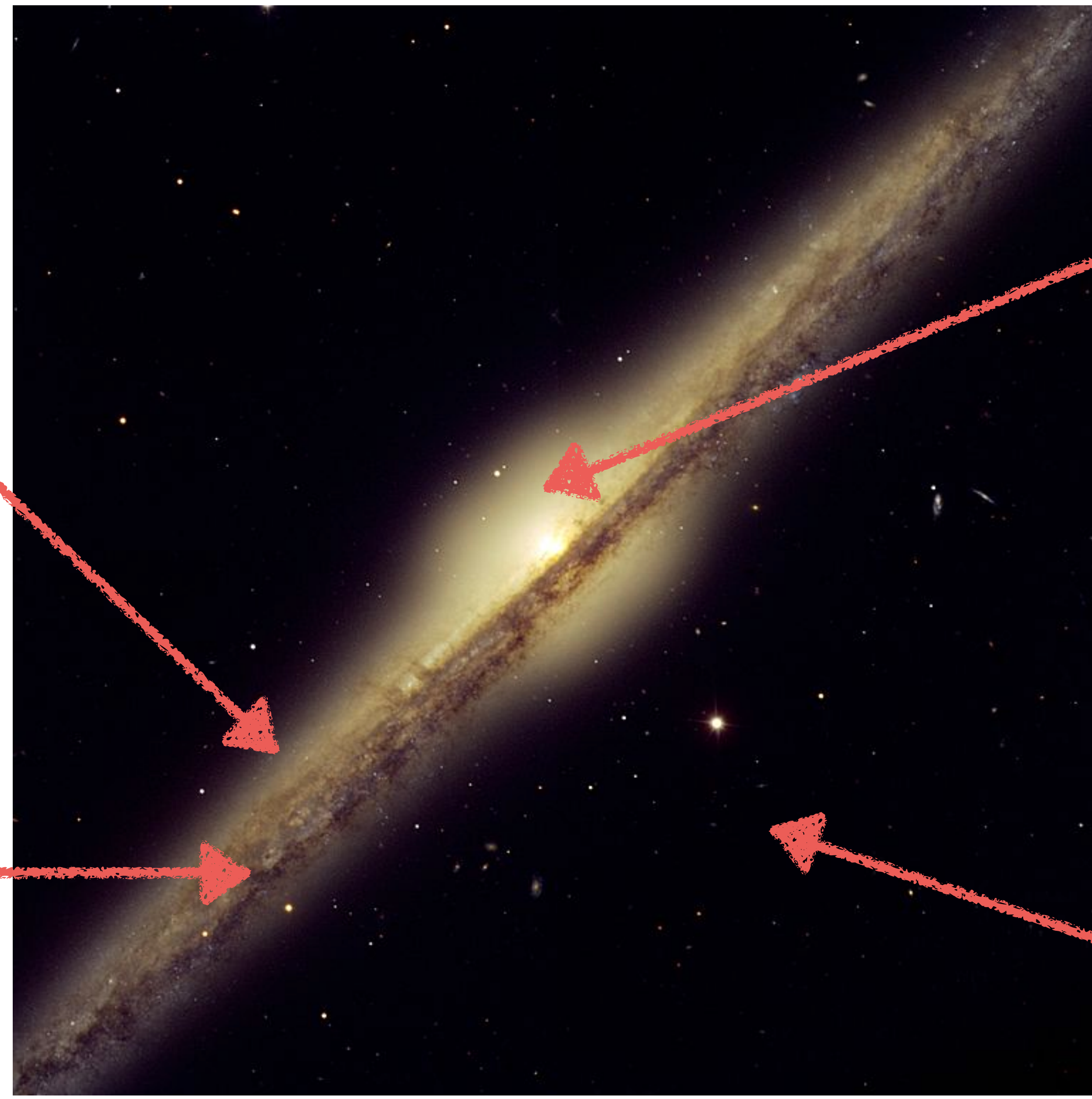
# Review time!

# Structure of galaxies

# Structure of galaxies: spiral like MW

Stellar disk (exponential, ~$10^{11}$ $M_{sun}$)

gas disk (diff. components, ~1/10 stellar disk)

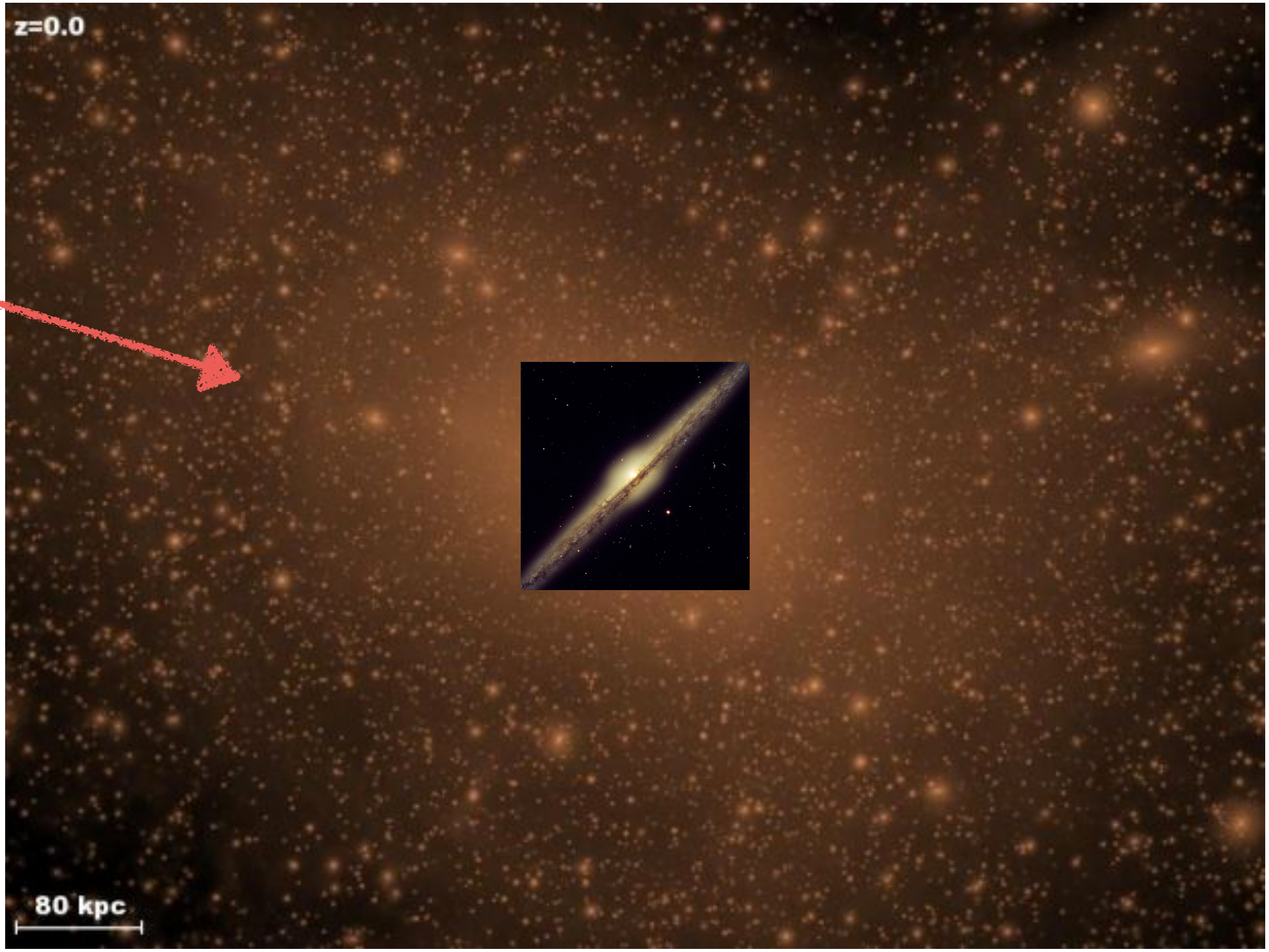


Bulge (Sersic, ~$10^{10}$ $M_{sun}$)

Stellar halo incl. globular clusters (power-law, ~$10^{9}$ $M_{sun}$)

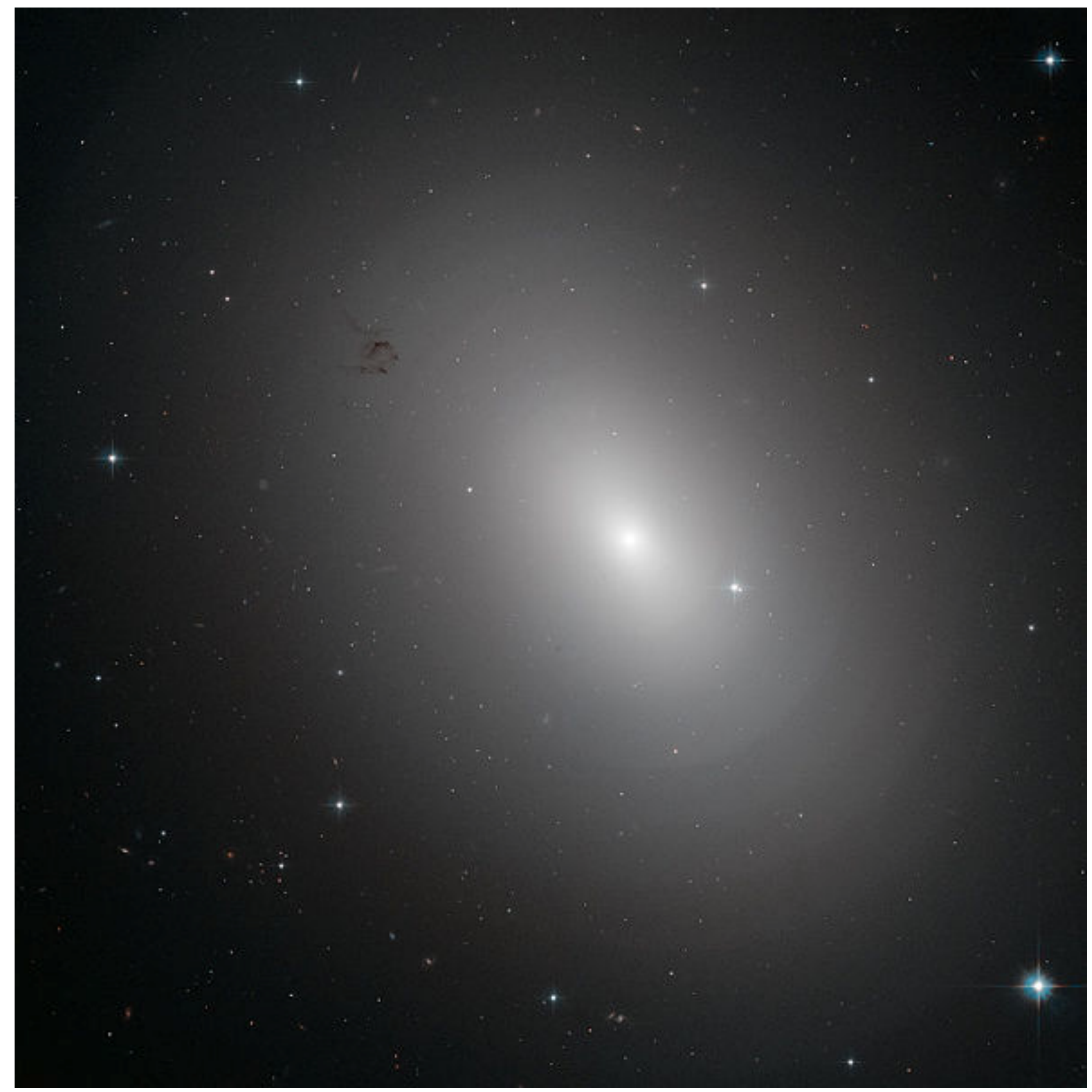
# Structure of galaxies: spiral like MW

Dark matter halo (NFW, ~10 times bigger in linear size than disk, ~$10^{12}$ $M_{sun}$)



# Structure of galaxies: elliptical galaxies

- “All bulge”
- Range in sizes from dwarf (<<MW) to massive ellipticals (>> MW)
- Found more typically in dense regions of the Universe



# Timescales

# Timescales

- Always to consider timescale over which effects happen to determine whether they are important or not and how to handle them
- For galaxies, fundamental timescales:
    - Age of the Universe (Hubble time $t_H$): 13.8 Gyr
        - Anything that takes >> $t_H$ is irrelevant for understanding galaxies *today*

# Timescales

- For galaxies, fundamental timescales:
  - Dynamical time: typical time for a star to orbit within a galaxy:

$$t_{\rm dyn} = \frac{2\pi\, R}{v}$$

    - Orbit of the Sun in the Milky Way: about 200 km/s at 8 kpc (1 km/s ~ 1 pc/Myr) ➞~ 250 Myr ~ $t_H$/40
    - Orbits in the outer halo: ~100 km/s at 100 kpc ➞~ 6 Gyr ~$t_H$
    - Related to mean density: $t_{\rm dyn} \approx 3\,(G\,\bar{\rho})^{-1/2}$

# Timescales

- For galaxies, fundamental timescales:
  - Relaxation time: timescale on which individual encounters are important:

$$t_{\rm relax} \approx \frac{N}{50\ \ln N} t_{\rm dyn}$$

  - Galaxies: N ~ $10^{11}$ —> ~$10^{10}$ Myr ~ $10^{6}$ $t_H$
  - Dense star clusters: N ~ $10^{5}$, $t_{dyn}$ ~ 1 Myr —> ~1 Gyr < $t_H$

# Timescales

- For galaxies, fundamental timescales:
  - Stellar evolution: few Myr (massive stars) over solar (~$t_H$) to many many $t_H$ (low-mass stars)
  - Cooling time: time to remove thermal energy from gas through radiative cooling
  - Gas depletion:

$$\frac{[\text{ gas-mass } (\sim \text{few } \times 10^9\ M_{sun})\ ]}{[\text{star-formation-rate } (\sim \text{few Msun / yr})\ ]} \sim 1 \text{ Gyr} < t_H$$

# Timescales

- For galaxies, fundamental timescales:
  - Chemical evolution time: timescale over which abundance of elements in the ISM increases, different for different elements due to different processes
  - Dynamical friction timescale: timescale over which a massive satellite spirals into a central galaxy

# Collisional vs. collisionless systems

# Galaxies as collisionless systems

- Saying that the gravitational force is smooth is the same as saying that collisions don't matter much to the orbits of stars

- To more quantitively determine whether collisions matter, we can compute the time necessary for close encounters to change the velocity by order unity

- Approximate treatment of galaxies allows a simple estimate to be made (see notes):

$$t_{\rm relax} = n_{\rm relax} t_{\rm cross} \approx \frac{N}{8 \ln N} \frac{R}{v} .$$

# Galaxies as collisionless systems

For galaxies, $N \approx 10^{11}$ and $t_{\rm cross} \approx 100$ Myr. Therefore,

```
trelax= 10.**11./8./numpy.log(10.**11)*100*u.Myr
print(trelax.to(10**10*u.Myr))
```

```
4.935164567082407 1e+10 Myr
```

- Therefore, collisions are only important on timescales >> the age of the Universe

- We can therefore usefully treat galaxies as smooth mass distributions

# Dense star clusters are collisional systems

- Dense star clusters have crossing times of ~1 Myr and ~$10^5$ stars

```
trelax= 10.**5./8./numpy.log(10.**5)*1*u.Myr
print(trelax.to(u.Gyr))
```

```
1.0857362047581296 Gyr
```

- Therefore, collisions in dense star clusters *are* important on timescales ~ the age of the Universe

- Dynamics of dense star clusters is much more complicated!

# The Poisson equation

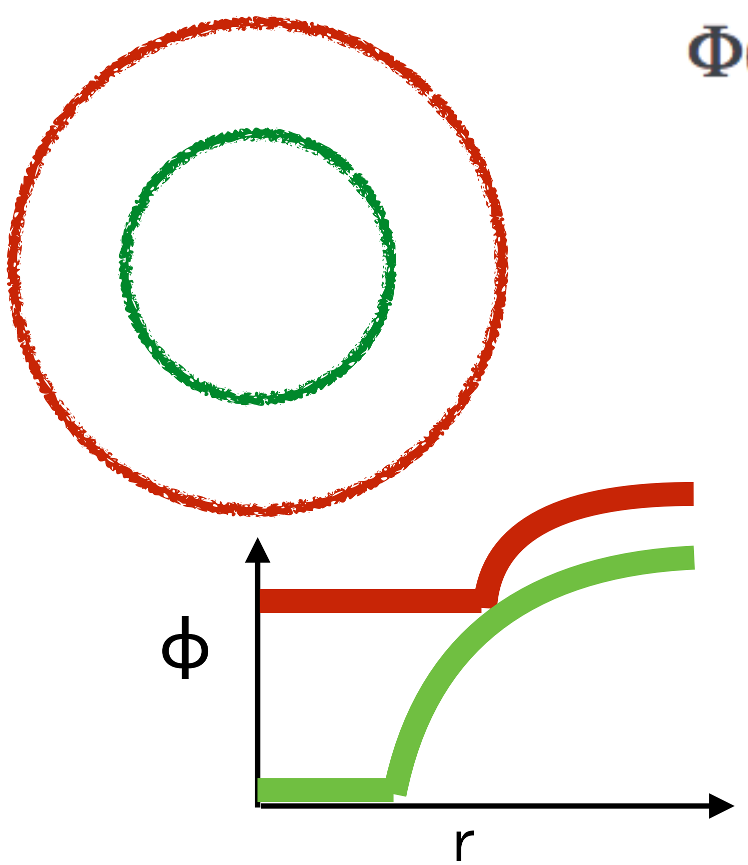
# Potential theory

- Gravitational *force* and gravitational *field*: force is F that goes in F=ma, field is g = F/m —> g = a for gravity (universality of free fall)
- Gravitational *potential* $$\mathbf{g}(\mathbf{x}) = -\boldsymbol{\nabla}\Phi,$$
- Poisson equation: $$\nabla^2\Phi = 4\pi G\rho.$$
- Poisson equation: partial differential equation, typically difficult to solve
- Strategies:
    - Simplify geometry of ρ (e.g., spherical)
    - Posit ϕ, see what ρ it implies

# Poisson equation for spherical systems

- Newton's shell theorem 1: inside spherical shell —> no force
- Newton's shell theorem 2: outside spherical shell —> as if point mass

$$\Phi(r) = -4\pi G \left[ \frac{1}{r} \int_0^r \mathrm{d}r' \, \rho(r') \, r'^2 + \int_r^\infty \mathrm{d}r' \, \rho(r') \, r' \right] .$$




- Mass outside current r has no influence on the acceleration
- For general mass distributions: effect of mass outside of r will typically be quite subtle (monopole has no effect)

# Circular velocity …

- Circular velocity —> mass *inside*

$$v^2 = -r\, g_r(r) = \frac{G\,M(< r)}{r}$$

- Dynamical time —> mean density *inside*

$$t_{\rm dyn} = \sqrt{\frac{3\pi}{G\,\bar{\rho}}}\,,$$

- Escape velocity —> potential & mass *outside*

$$v_{\rm esc} = \sqrt{2[\Phi(\infty) - \Phi(r)]}\,,$$

# Orbits

# Classical mechanics

- Equations of motion: Newton's second law: **F** = m **a**
- Lagrange formalism: easier to derive EOM in different coordinates; *Lagrangian* L = K-V = $|\mathbf{v}|^2/2-\phi(\mathbf{x})$
- Lagrange's equation for *any coordinate system:*

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\mathcal{L}}{\partial\dot{\mathbf{q}}}\right)-\frac{\partial\mathcal{L}}{\partial\mathbf{q}}=0, \qquad \mathbf{p}\equiv\left(\frac{\partial\mathcal{L}}{\partial\dot{\mathbf{q}}}\right)_{\mathbf{q},t}.$$

- Hamiltonian (static potential)

$$H(\mathbf{x},\mathbf{p})=\mathbf{p}\cdot\dot{\mathbf{x}}-\mathcal{L}=\frac{p^2}{2m}+V(\mathbf{x});$$

- Hamilton's equations:

$$\dot{\mathbf{q}}=\frac{\partial H}{\partial\mathbf{p}} \quad ; \quad \dot{\mathbf{p}}=-\frac{\partial H}{\partial\mathbf{q}}.$$

# Integrals of motion

- Integrals of motion: functions $I(\mathbf{x},\mathbf{v})$ that are conserved along an orbit
- Useful, because constrain the orbit to a lower-dimensional subspace
- Typically derived from inspection of Lagrangian: Lagrangian doesn’t depend explicitly on coordinate *r* —> associated momentum is conserved
- Important examples:
    - Time-independent potential —> energy E conserved
    - Spherical potential —> angular momentum vector **L** conserved
    - Axisymmetric potential —> *z*-component of **L**, $L_z$, conserved
    - Potential rotating at constant pattern speed $\Omega_b$—> Jacobi integral $E_J = E - \Omega_b L_z$ conserved

# Third integral

- Many orbits in galactic potential also conserve a third integral for which no explicit expression exists, but known from surface of section

- Orbits which have three integrals of motion are *regular*: can be usefully thought of as consisting of three coupled oscillations:

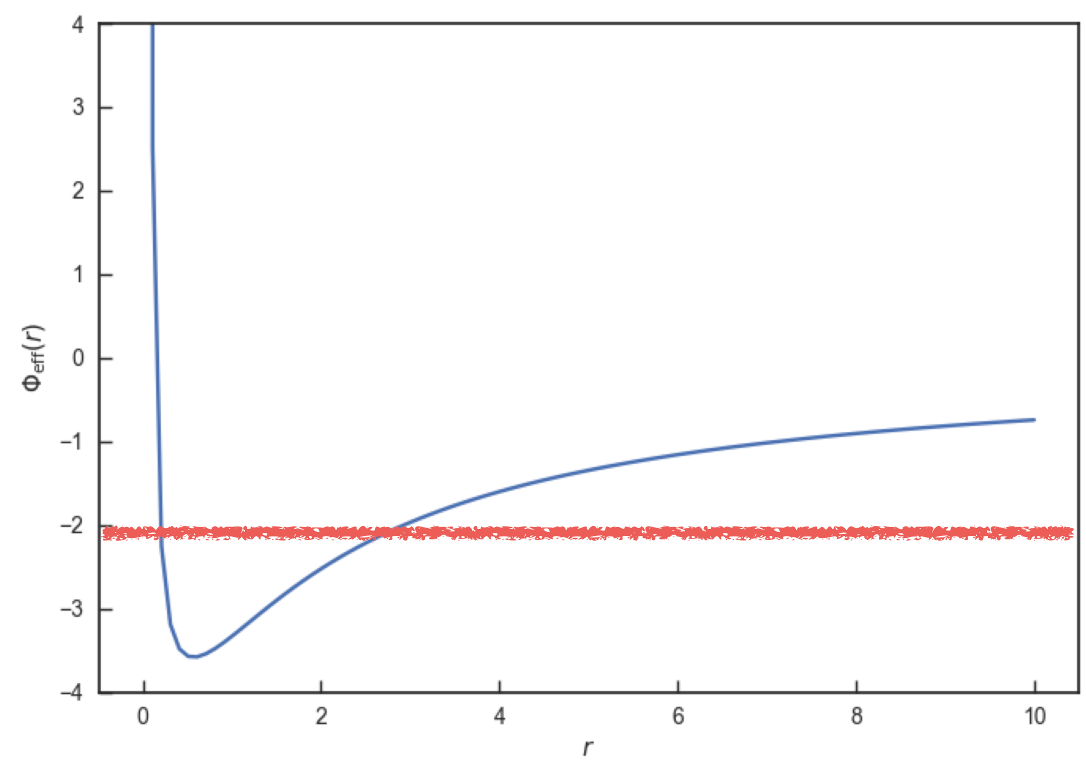
    - Circular motion of around the center within an ‘average’ orbital plane

    - Radial oscillation wrt the circular orbit

    - Vertical oscillation wrt the ‘average’ orbital plane

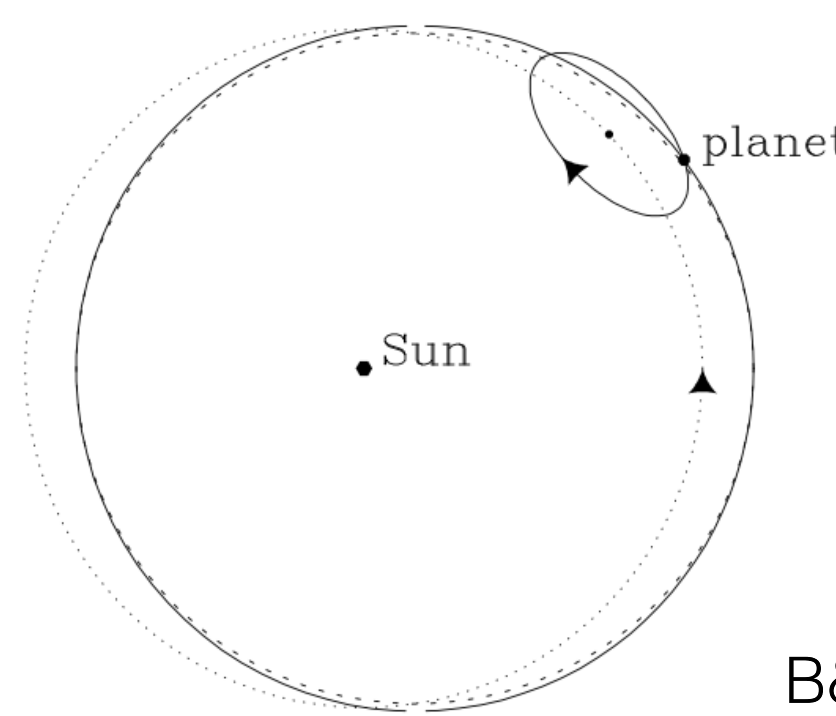
# Epicycle approximation

- Orbits cannot be solved analytically for any realistic galactic potential
- Significantly impedes progress in understanding galactic dynamics: compare to solar system dynamics where analytical solutions of the dynamics have for centuries fueled advances in celestial mechanics
- Very useful general approximation: epicycle approximation: expand effective potential up to quadratic terms near minimum —> harmonic oscillator which can be solved analytically




$$R(t) - R_g = X\ \cos(\kappa\, t + \alpha)\,,$$

$$R_g\ (\phi(t) - \Omega\, t - \phi_0) = -2\, \frac{\Omega}{\kappa}\, X\ \sin(\kappa\, t + \alpha)\,.$$

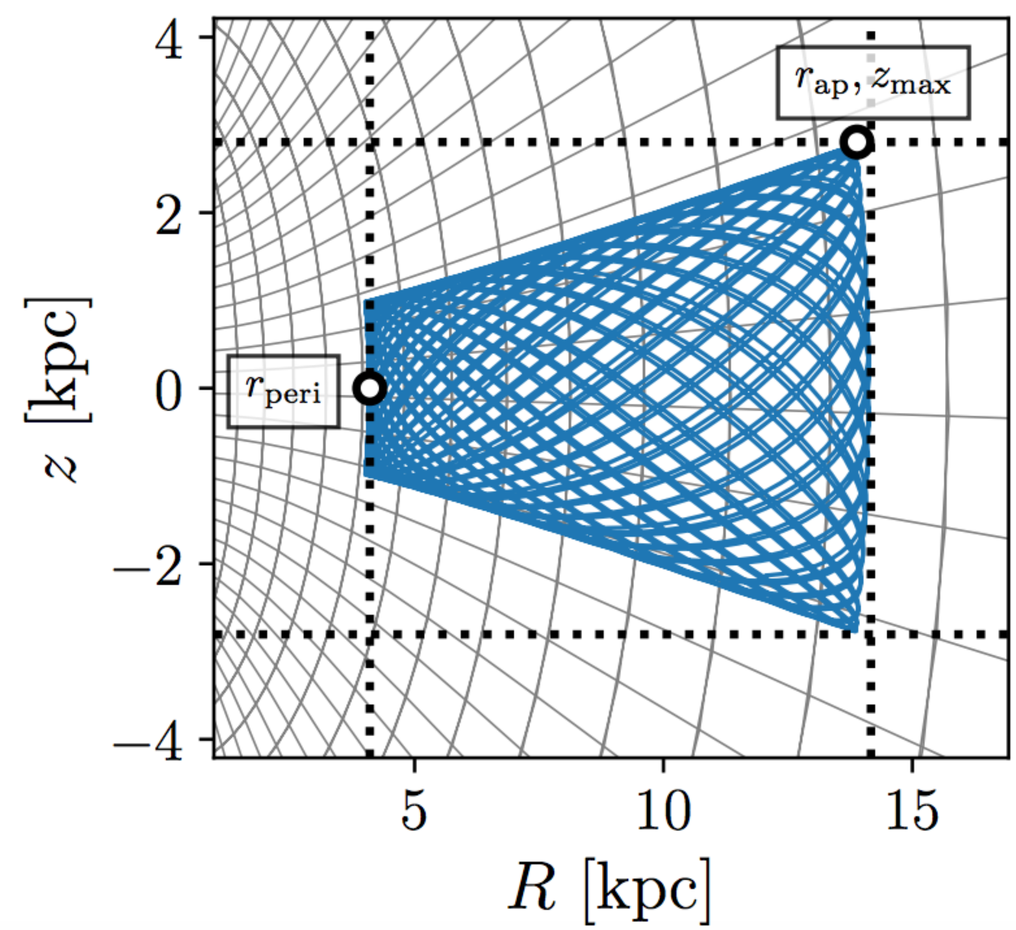



B&T Fig. 3.7

# Properties of orbits

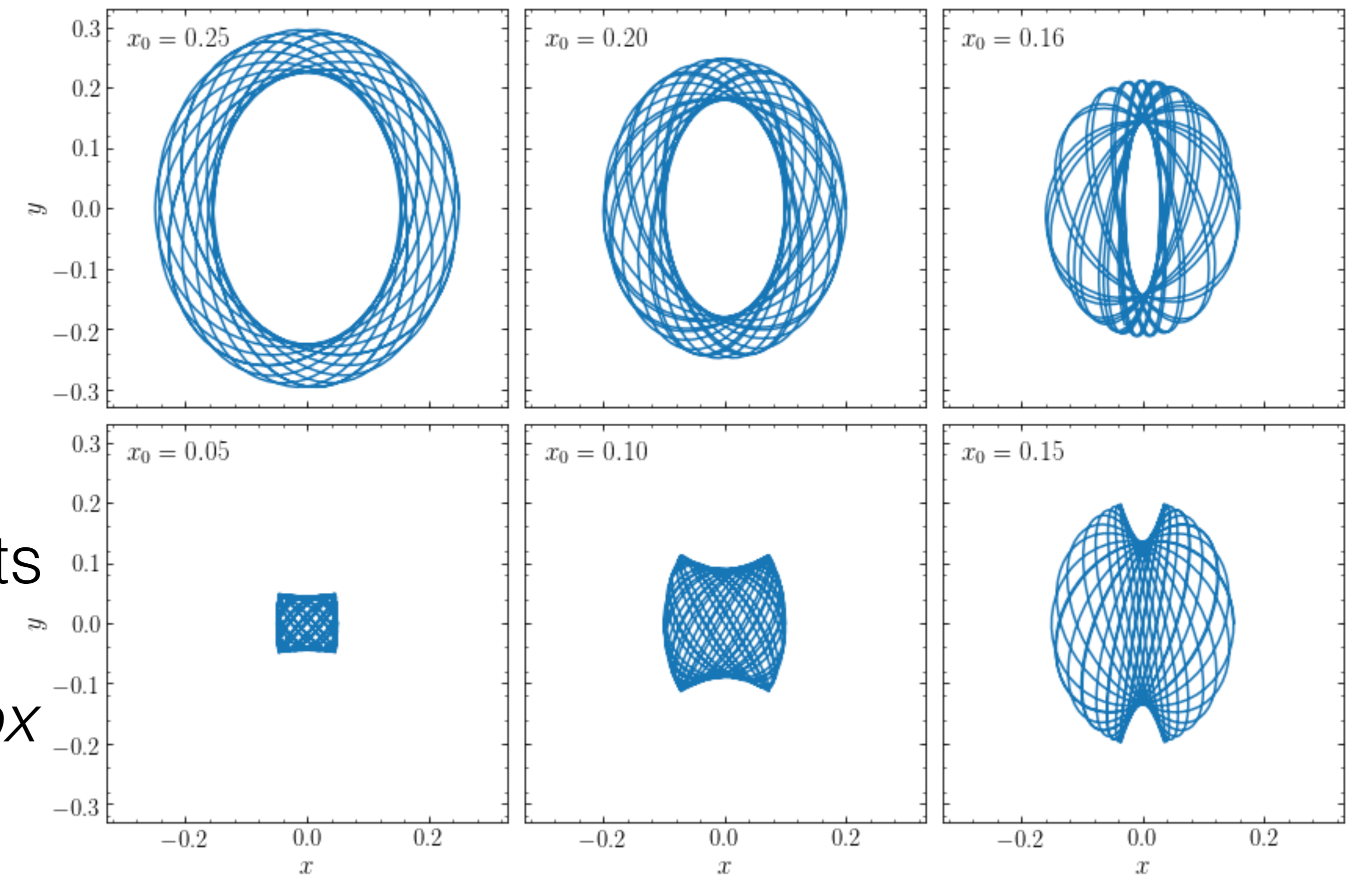
- pericenter, apocenter: closest and furthest point from center
- Guiding-center radius: radius of circular orbit with same angular momentum
- mean radius: average of peri- and apocenter
- eccentricity: $(r_{ap}-r_{peri})/(r_{ap}+r_{peri})$

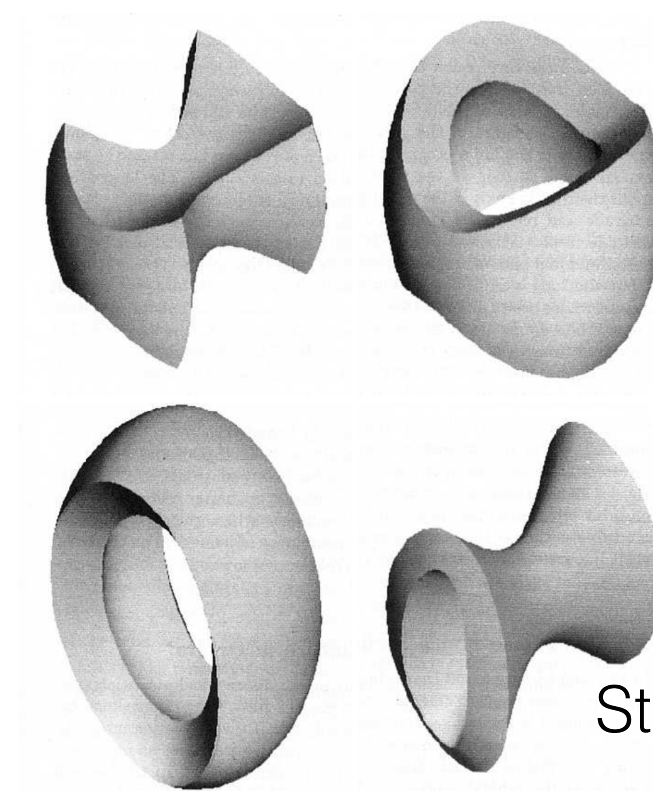
Mackereth et al. (2018)



# Types of orbits

- We've encountered various types of orbits
- Axisymmetric potentials: all *loop* orbits
- Non-axisymmetric potentials have *box* orbits: no definite sense of rotation, can get arbitrarily close to r=0
- Triaxial potentials: different types of loop and box orbits
- Rotation potentials (like bars) have even more complex orbits (bananas!?)



Statler (1987)

# Equilibrium

# Equilibrium of galaxies

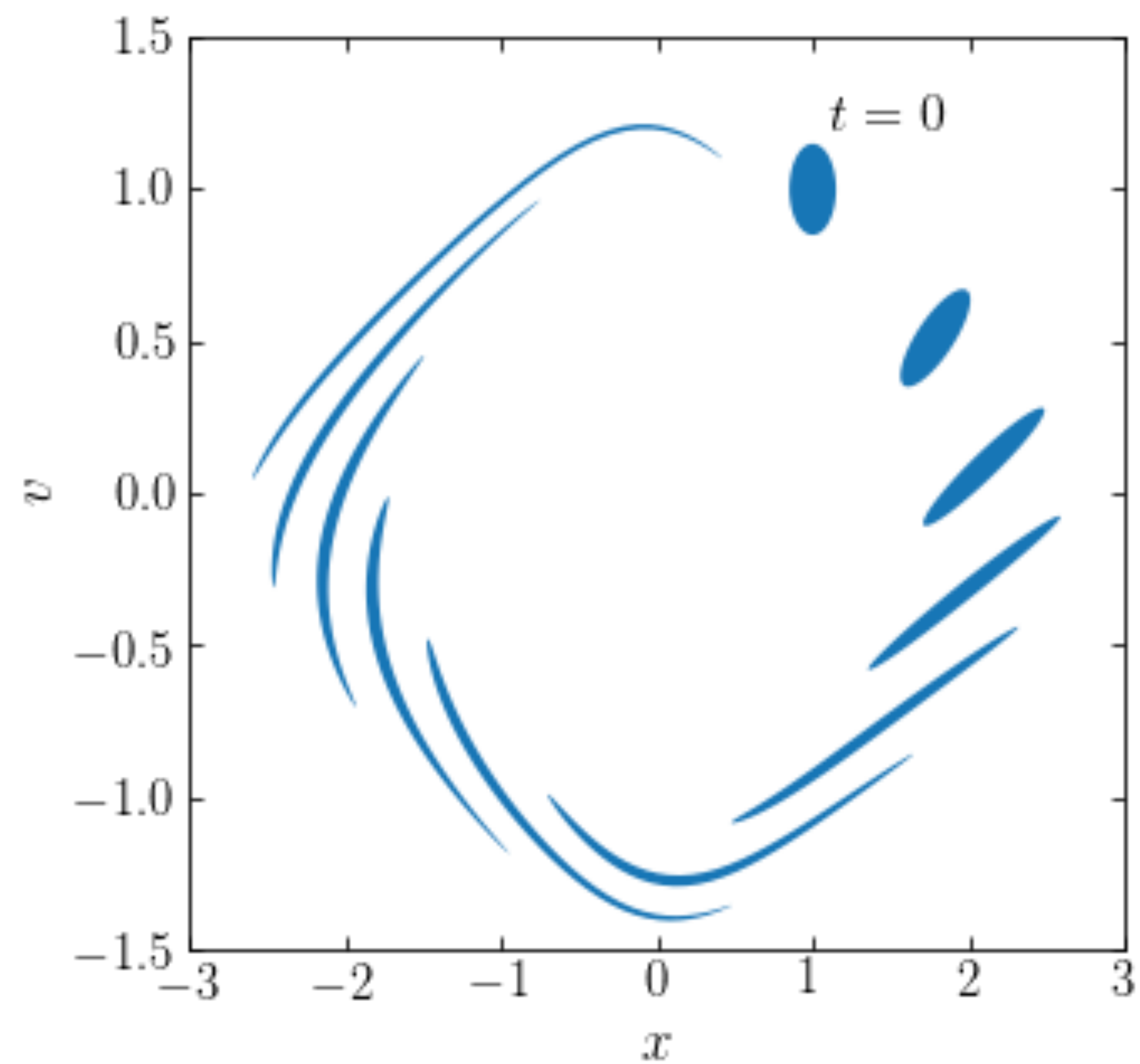
- Evolution of galaxies and dark matter described by the *collisionless Boltzmann equation* for the evolution of *phase-space density f:*

$$\frac{\partial f(\mathbf{x},\mathbf{v},t)}{\partial t} + \dot{\mathbf{x}}\,\frac{\partial f(\mathbf{x},\mathbf{v},t)}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{x}}\,\frac{\partial f(\mathbf{x},\mathbf{v},t)}{\partial \mathbf{v}} = 0\,,$$

- Describes dynamics within galaxies and of dark matter perturbation growth in the Universe
- Equilibrium: d f / d t = 0
- Solutions:
    - Any function of the integrals of motion (Jeans theorem)
    - Constraints from taking moments: Jeans equations of which we have seen a few varieties (spherical with important role of radial anisotropy, cylindrical)
    - Virial theorem is another consequence of the CBE

# Liouville theorem

$$\frac{\mathrm{d}f(\mathbf{x},\mathbf{v})}{\mathrm{d}t} = \frac{\partial f(\mathbf{x},\mathbf{v})}{\partial t} + \dot{\mathbf{x}}\,\frac{\partial f(\mathbf{x},\mathbf{v})}{\partial \mathbf{x}} + \dot{\mathbf{v}}\,\frac{\partial f(\mathbf{x},\mathbf{v})}{\partial \mathbf{v}} = 0\,,$$

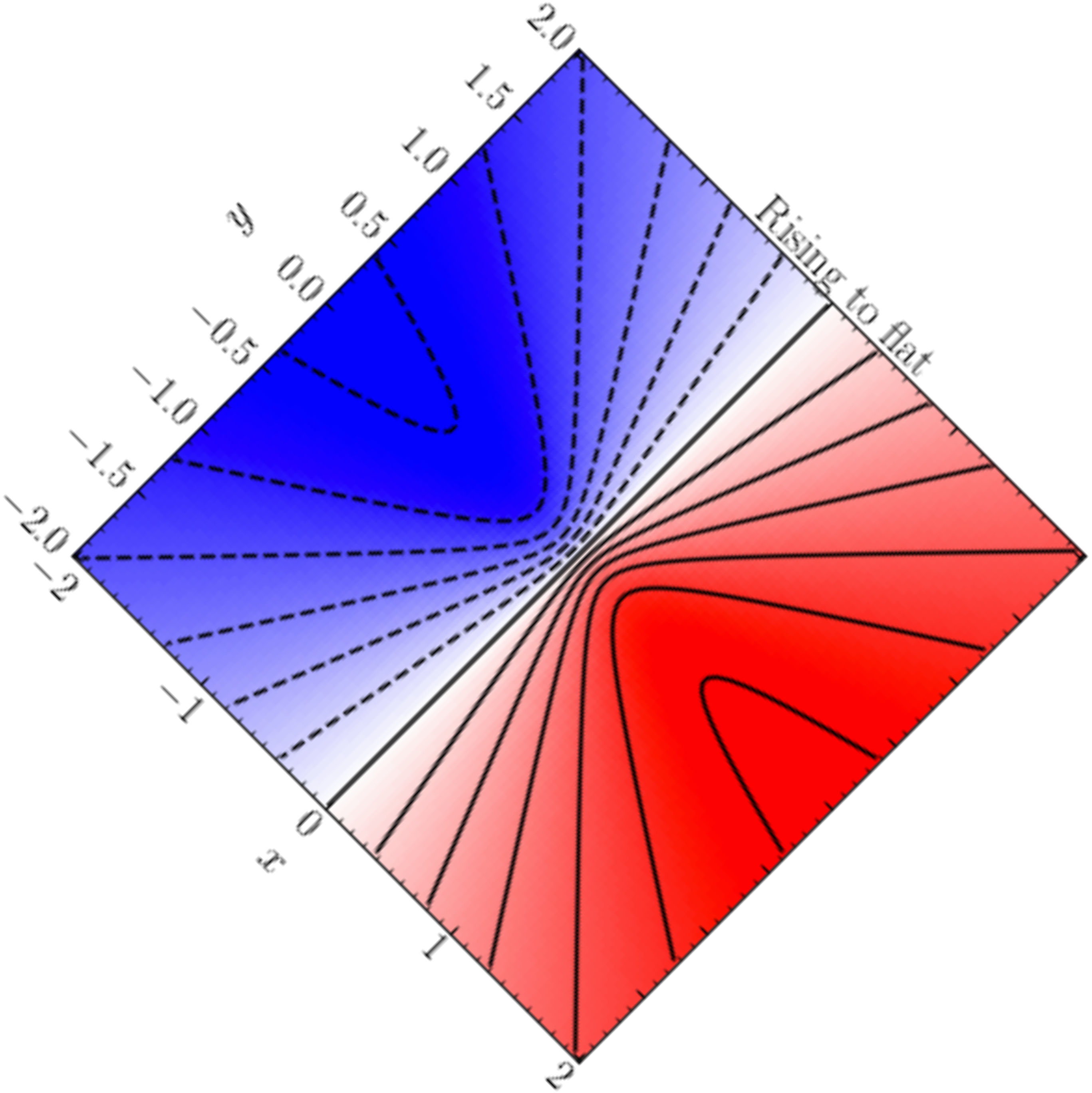


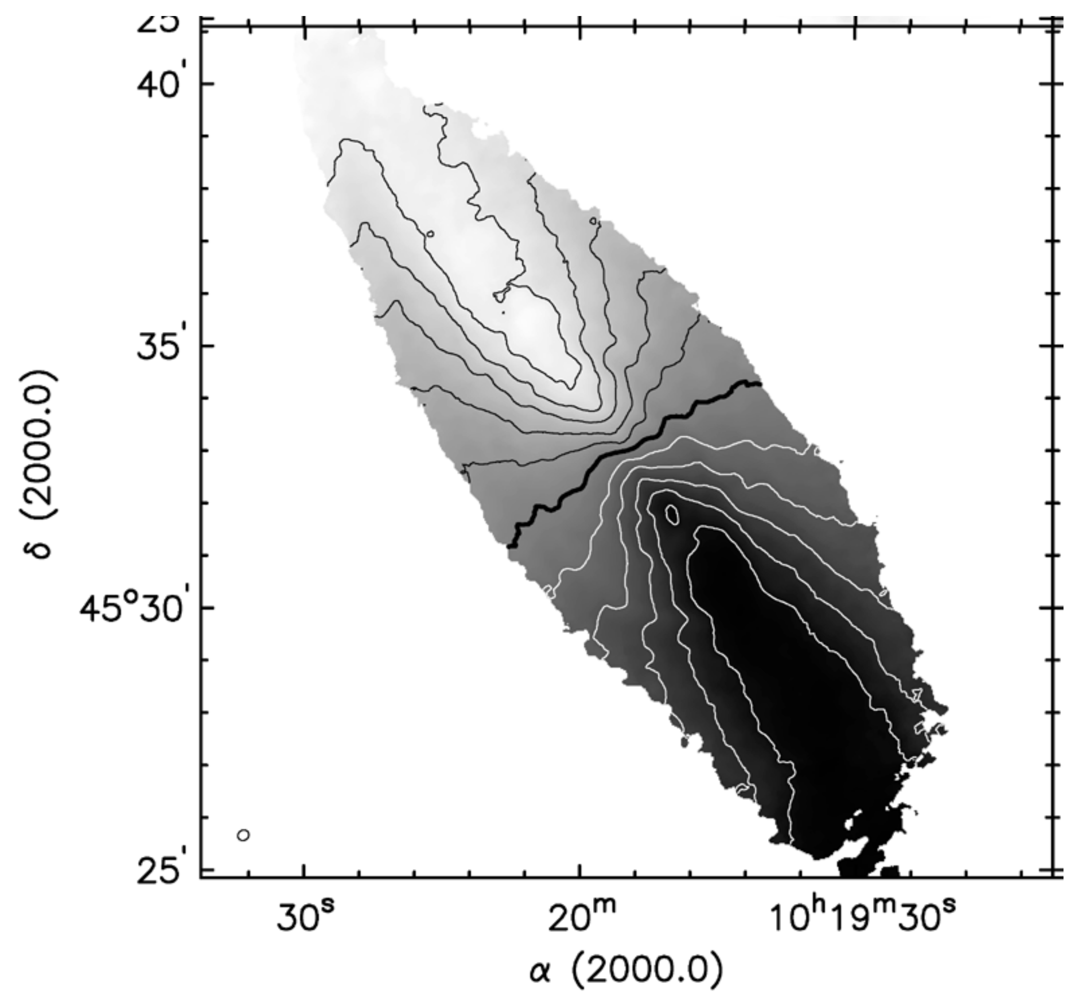
# Dark matter

# Evidence for dark matter

- Hope you appreciate after this class that why we nowadays believe there is DM in the Universe largely comes from galactic dynamics observations and theory like what we discussed in class
- Picture is more complicated than cartoon ‘flat rotation curve’ picture
- Crucial developments:
    - Zwicky’s application of the viral theorem to velocities of galaxies in the Coma cluster
    - Kahn & Woltjer’s Local Group timing argument
    - Rubin et al. rotation curves —> importance of going to large radii and taking into account mass-to-light ratio
- Modern (~last two decades) evidence: CMB, weak lensing [but case incontrovertible long before]
- Current controversies: ‘missing satellites’ (where are the low-mass DM subhalos), cusp-vs-core (inner density profile predicted to be 1/r, but observed const. density), too-big-too-fail (inner density of large satellites observed to be lower than predicted)

# Reading velocity fields



# Growth of structure in the Universe

# Formation of galaxies

- In our cosmological paradigm: dark matter (DM) is ~5x more abundant than ordinary matter (‘baryonic matter’, ‘standard model matter’)
- Dark energy (DE) is presently more about 3x more abundant than DM, but
  - Simplest DE model does not cluster, only affects overall expansion of the Universe
  - Until recently, DE << DM because DM grows as $(1+z)^3$ —> much of galaxy formation in DM-dominated, flat Universe —> Einstein—de Sitter model
- DM only interacts gravitationally, so can start clustering at $z >> 1000$ (recombination), while baryons are coupled to photons until recombination and cannot cluster much until then
- Formation of galaxies can be thought of as: DM clusters and forms halos, baryons (gas) collects in halos, sinks to the center through EM energy loss, cools and forms stars

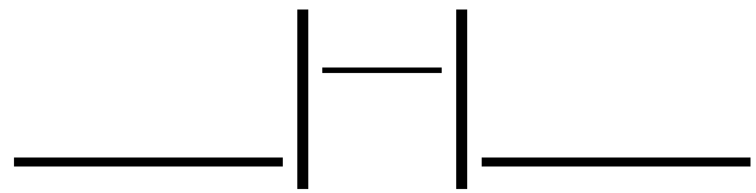
# Evolution of small perturbations

- For DM, derived from collisionless Boltzmann equation!
- DM: $$\frac{\partial^2 \delta}{\partial t^2} + 2\,\frac{\dot{a}}{a}\,\frac{\partial \delta}{\partial t} = 4\pi\, G\,\overline{\rho}\,\delta\,.$$
- Baryons w/ equation of state relating pressure, density, and entropy:

$$\frac{\partial^2 \delta}{\partial t^2} + 2\,\frac{\dot{a}}{a}\,\frac{\partial \delta}{\partial t} = 4\pi\, G\,\overline{\rho}\,\delta + \frac{c_s^2}{a^2}\,\nabla^2 \delta$$

- Simplest in Einstein—de-Sitter Universe (flat, all matter): pressures growth δ(a) ~ a
- Baryons: growth similar down to Jeans scale, growth inhibit on smaller scales

# Halo formation



- Halos form when $\delta > 1$: over densities become non-linear, gravitationally collapse and form a bound, virialized object

- Spherical top-hat collapse: halo forms when *linear* over density reaches $\delta > \delta_c$ =~ 1.686

- Collisionless relaxation causes a virialized halo to form: simple considerations using virial theorem + spherical collapse give

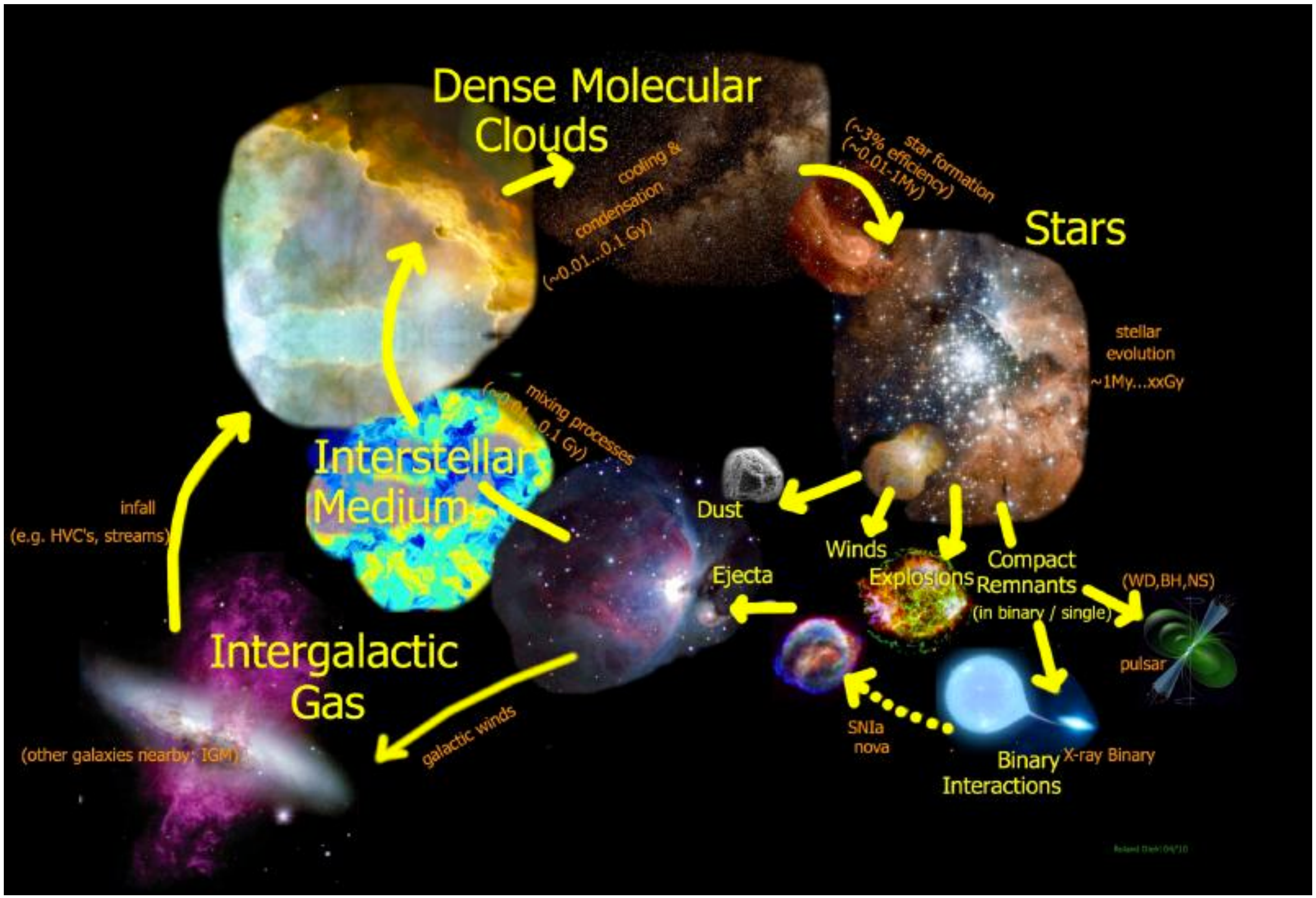
virial radius: $$r_v = \frac{r_{\max}}{2}$$

physical overdensity: $$\Delta_v = 18\,\pi^2\,\Omega_{m,i}^{-1}\,.$$

~ 178 >> 1.686

# Galactic chemical evolution

# Complex cycle of star formation and galactic evolution

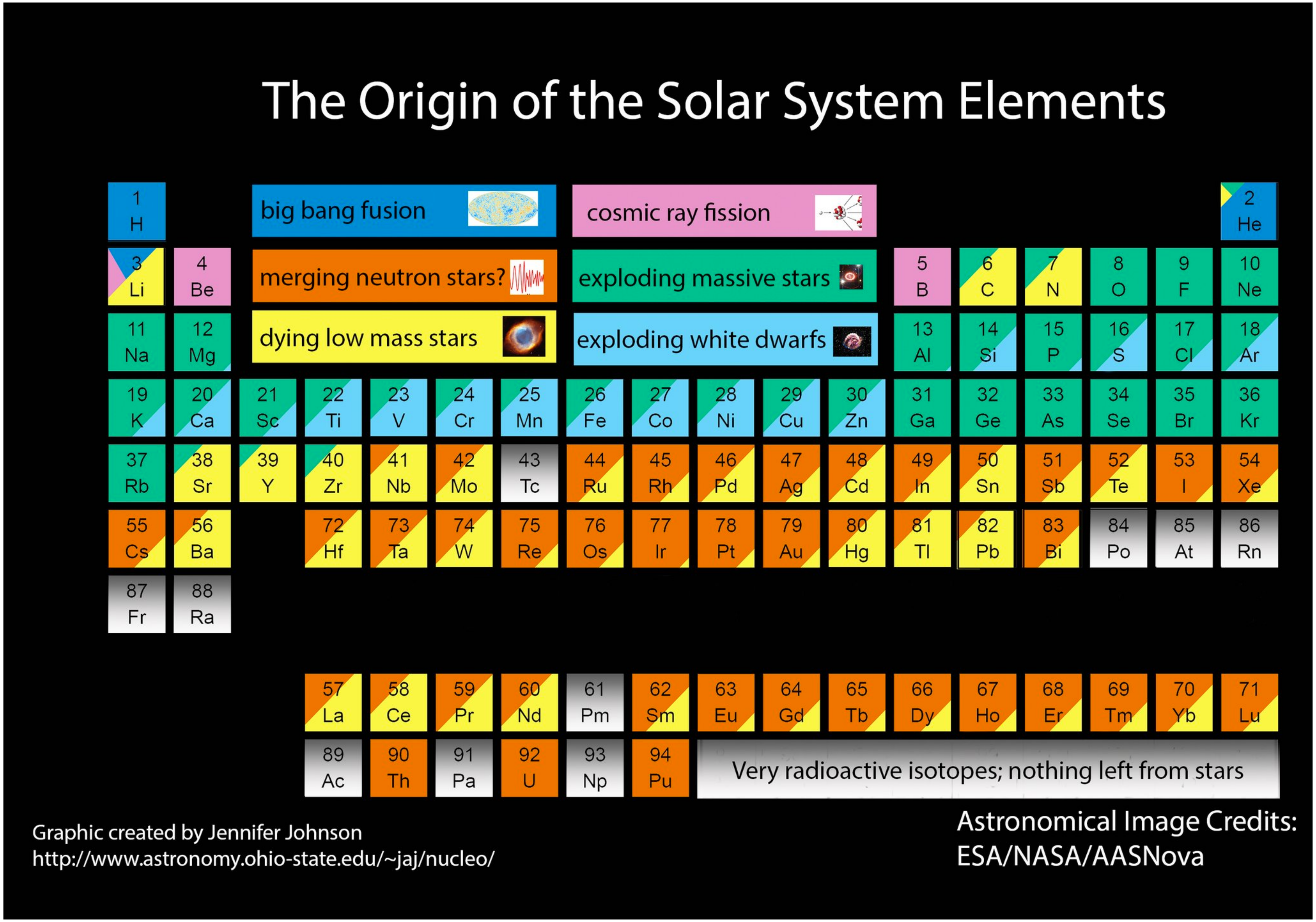



Credit: Roland Diehl (MPE, Germany)

# Simple chemical evolution models

- Closed-box model:
  - All gas present at start
  - Metallicity: set solely by
    (a) effective yield and (b) gas fraction, no dependence on anything else
  - Leads to wide metallicity distribution, with many low- and high-metallicity stars
- Leaky-box:
  - All gas present at start, but some gas expelled
  - Like closed box, just lower effective yield
- Accreting-box:
  - Gas that fuels star formation is slowly accreted to keep SFR ~ constant
  - Most stars formed near equilibrium metallicity, few low-metallicity stars and cut at high metallicity

# Chemical evolution of different elements



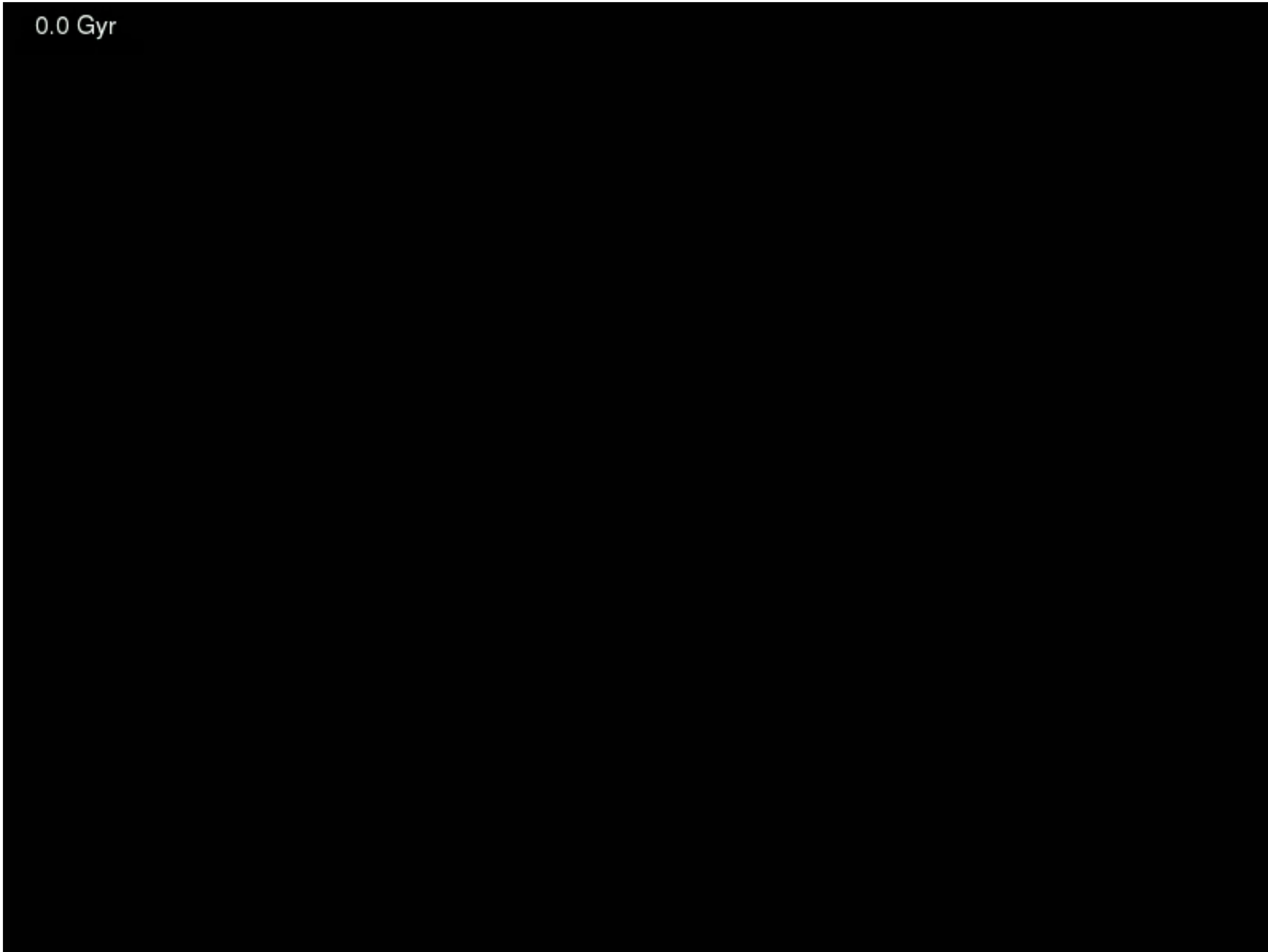
# Chemical evolution of different elements

- Chemical evolution set by *channels* that contribute to element
- Main channels:
    - Type II supernovae: prompt (~10 Myr after star formation)
    - Type Ia supernovae: long tail toward Gyr delays
    - Winds from AGB stars: timescale somewhere between type II and Ia
    - Neutron-star mergers (long delay tail), exotic massive star scenarios (prompt) —> still quite unclear!

# Galaxy formation and evolution

# Zero-th order galaxy formation and evolution

- Dark matter starts gravitational collapse at z >> 1000
- After baryon-photon decoupling at z ~ 1000, baryons start to get attracted to dark matter over densities
- Overdensities become so massive that they decouple from the Universe's expansion —> formation of dark matter halos; earliest at z ~ tens
- Baryons gather in large dark-matter halos, cool to form cold disks near the center
- Successive generations of star formation deplete and enrich gas in the disk, pristine gas is continuously accreted
- Mergers bring in stars (and SMBH) from other galaxies —> stellar halo, major mergers of disks create elliptical galaxies
- Galaxies are marginally stable (Q~1): Bars, spiral structure, perturbations from satellites mix up stars in galaxies, drive gas flows that may, e.g., fuel AGN activity and grow SMBH at the center of galaxies




Credit: Greg Stinson, MUGS (http://mugs.mcmaster.ca/)

That’s it, thanks for listening!