AST1420 "Galactic Structure and Dynamics" Problem Set 2

Due on Nov. 5 at the start of class

Most of the exercises in this problem set must be solved on a computer and the best way to hand in the problem set is as an `ipython notebook`. Rather than sending me the notebook, you can upload it to `GitHub`, which will automatically render the notebook. Rather than starting a repository for a single notebook, you can upload your notebook as a `gist`, which are version-controlled snippets of code that can optionally be made private. If you want to make further changes, you can clone your gist in a separate directory and use it as you would any other git repository. *Please re-run the entire notebook (with `Cell > Run All`) after re-starting the notebook kernel before uploading it*; this will make sure that the input and output are fully consistent.

If you are unfamiliar with notebooks, you can also send in a traditional write-up (in LaTeX), but you also need to send in well-commented code for how you solved the problems. Thus, notebooks are strongly preferred :-)

Problem 1: Dark matter and scaling relations in disk galaxies.

In one of the recent Q&A sessions, we played with rotation curves from the SPARC compilation (Lelli et al. 2016) and decomposed them into disk and halo contributions. Let's see what else we can learn about dark matter and galaxy structure from these data!

Important note: You can use the functions from the class notebook to load the and process the data, but be aware that the table entries as read by the code do *not* appear to have proper `astropy` units set. This means that when you naively pass them to, e.g., `galpy` functions, `galpy` will interpret the input as having no units. You can get around this issue by explicitly casting the table to the units that you want, for example, `data['R'].to(u.kpc)`. If you use `galpy` for this problem (which is probably useful...), it may also help to add a configuration file[1] with `astropy-units = True` (this is not the default) to the directory in which you are working. That way, `galpy` functions will always return `astropy` Quantities with units attached.

(a) When we did the exercise in class, we used disk rotation curves that were unconstrained by other data. But we in fact have surface-brightness measurements for all of the galaxies in the sample (included in the table as `SBdisk`), which we can use to determine an appropriate disk rotation curve and its contribution to the total rotation curve. For the three galaxies that we studied in class, re-fit the rotation curve as a razor-thin disk plus NFW halo, but first finding good disk parameters by fitting the surface brightness profile (assume $M/L = 1$ as is appropriate in the mid-IR used in the SPARC compilation). You can just fit by hand and determine parameters that roughly fit by eye.

[1] See `https://docs.galpy.org/en/latest/installation.html#configuration-file` (note that some PDF viewers do not properly work when clicking this link, so you may have to copy it).

(b) What do you learn about the dark matter contribution in these galaxies?

(c) The Tully-Fisher relation is a famous scaling relation for disk galaxies that relates a galaxies asymptotic velocity V_∞ to its luminosity. It can be used, for example, to obtain distances to galaxies, because the rotation velocity can be measured relatively easily and the luminosity obtained from the relation can then be combined with the observed magnitude to obtain the distance. It's also a useful constraint on galaxy formation models. Let's use the bulgeless SPARC galaxies to measure the Tully-Fisher relation ourselves! To obtain a quantitative determination of V_∞, we can fit the rotation curves with a gravitational potential of the form

$$\phi(r) = \frac{v_0^2}{2}\,\ln\left(r^2 + r_0^2\right)\,, \tag{1}$$

with two free parameters, v_0 and r_0. Determine the circular velocity as a function of radius $v_c(r)$ for this potential. What is V_∞ in terms of v_0 and r_0?

(d) Now fit this model to all bulgeless SPARC galaxies. Fit the model rotation velocity by minimizing χ^2 defined as

$$\chi^2 = \sum_i \frac{\left[v_c(\mathtt{R}_i) - \mathtt{Vobs}_i\right]^2}{\mathtt{e\_Vobs}_i^2}\,, \tag{2}$$

where i indexes the N data points for each galaxy $\{\mathtt{R}_i, \mathtt{Vobs}_i, \mathtt{e\_Vobs}_i\}$. You can use `scipy` optimization functions for this. Compare the model fit to the observed rotation curve for the three galaxies from part (a) and do the fit for all 143 bulgeless galaxies.

(d) Now derive the Tully-Fisher relation by computing the total luminosity of each galaxy and plotting V_∞ versus the total luminosity. Discuss what you see. The Tully-Fisher relation is generally well fit as $L \propto V_\infty^\alpha$; if you fit this form to your obtained relation, what value for α do you get (you can fit without taking errors into account). What α do you get if you only include $L > 10^{10}\,L_\odot$ galaxies? Which of these two determinations do you think is more accurate and why?

Problem 2: The zero-velocity curve.

When we looked at orbits in disk galaxies, we discussed the zero-velocity curve: the curve in the meridional plane (R, z) where $v_R = v_z = 0$. Let's explore this important curve further!

(a) For the example orbit in Chapter 10.1, we showed the zero-velocity curve as a constant energy contour, but we could also explicitly compute z as a function of (R, E, L_z) on the zero-velocity curve by demanding that $v_R = v_z = 0$. Do this for the first example orbit in Chapter 10.1 and compare to the zero-velocity curve that you get from the contouring method. (note that you may not be able to solve for z analytically and might have to resort to numerical methods...)

(b) In the notes, we discussed how orbits touch the zero-velocity curve at four points, but there are orbits at a given (E, L_z) that only touch the zero-velocity curve at *two* points. These orbits are called the thin tubes, because in 3D they look like a tube (that's thin!). Devise an algorithm to find this orbit for a given (E, L_z) and apply it to the (E, L_z) of the example orbit in (a) [there are various, qualitatively different ways to do this; explain why your algorithm works in general]. Show the orbit in (R, z) and (x, y, z)? Discuss what you think is interesting about this orbit.