Testing Robustness of Bayesian Model Comparison In Cosmological Analysis

Seo Won Yi[1, 2, 3], Tanveer Karim[1], Joshua Speagle[1, 3]

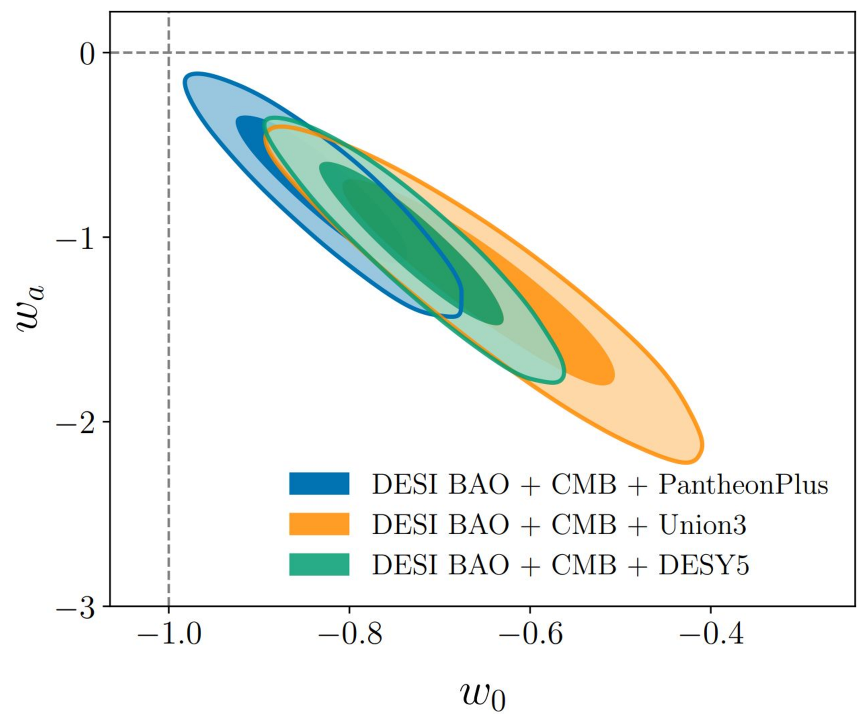
[1]David A. Dunlap Department of Astronomy & Astrophysics, University of Toronto
[2]Department of Computer Science, University of Toronto
[3]Department of Statistical Science, University of Toronto

Introduction

One of the major difficulties in cosmological analysis is finding a more accurate model to explain the behavior of the universe. Recent discoveries often reveal discrepancies with standard models, highlighting the field's vast potential for exploration and refinement.

This project examines the robustness of conventional model comparison methods used in cosmological analysis, emphasizing the importance of critically evaluating these methods rather than relying on them unquestioningly.

Motivation



- There exists discrepancy between standard values of Dark Energy parameters and observed findings
- The dotted lines represent the standard values.

Models

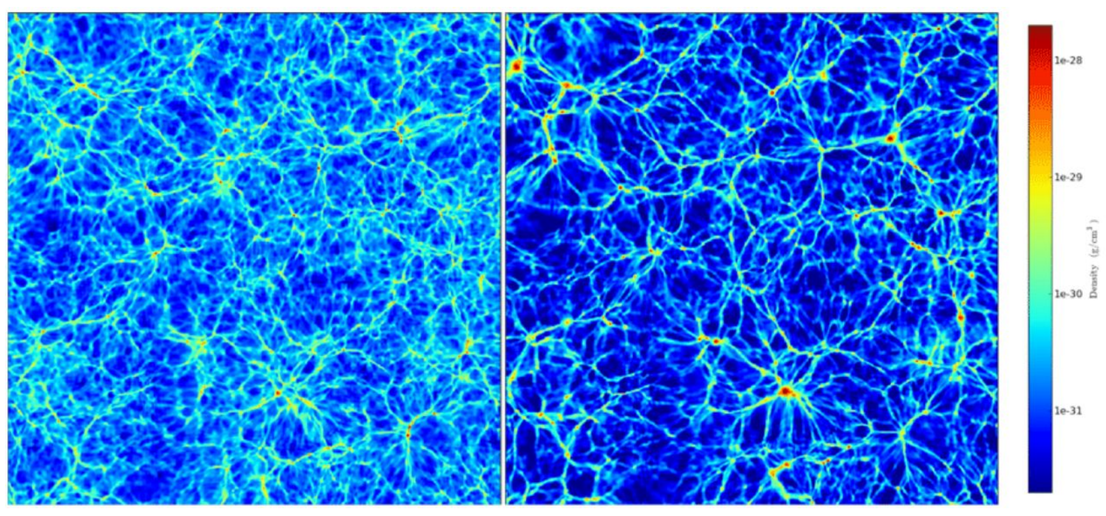


Illustration: Courtesy of Shankar Agarwal and Hume Feldman, University of Kansas

We compare the standard **Model 1**, which assumes a fixed neutrino mass, with **Model 2**, where the neutrino mass is treated as a free parameter.

A larger neutrino mass leads to less dense galaxies, as illustrated on the left side of the figure.

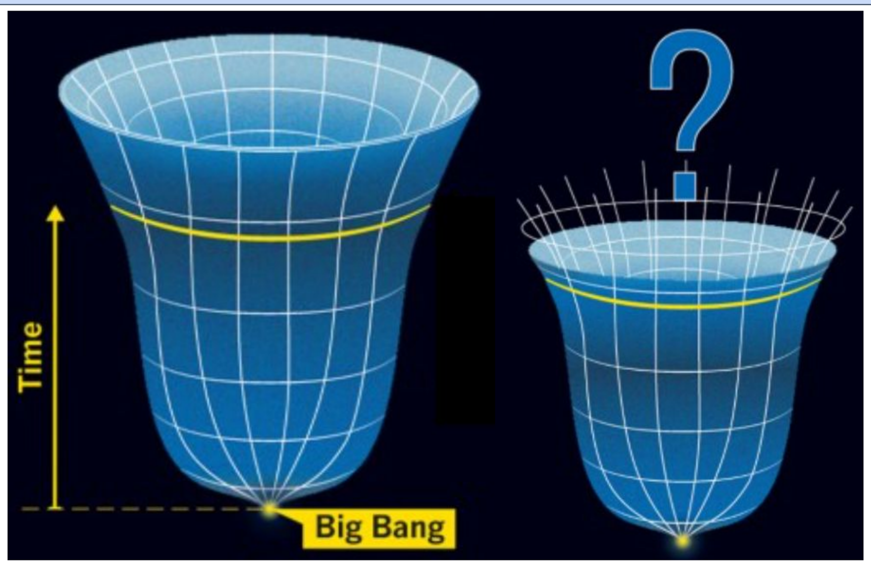


Illustration: Battersby, S. (2016). Dark energy: Staring into darkness. *Nature*

We compare the standard **Model 1**, which follows the ΛCDM framework, with **Model 3**, where dynamic dark energy is taken into account.

Dynamic dark energy influences the Universe's expansion by modifying the expansion rate.

Analysis

Deviance Information Criterion:

$$\mathrm{DIC} = \mathrm{Deviance}(\hat{\theta}) + p_D$$

where ***Deviance*** is the log-likelihood of model evaluated at posterior mean of parameters. $\boldsymbol{P_D}$ is the effective number of parameters.

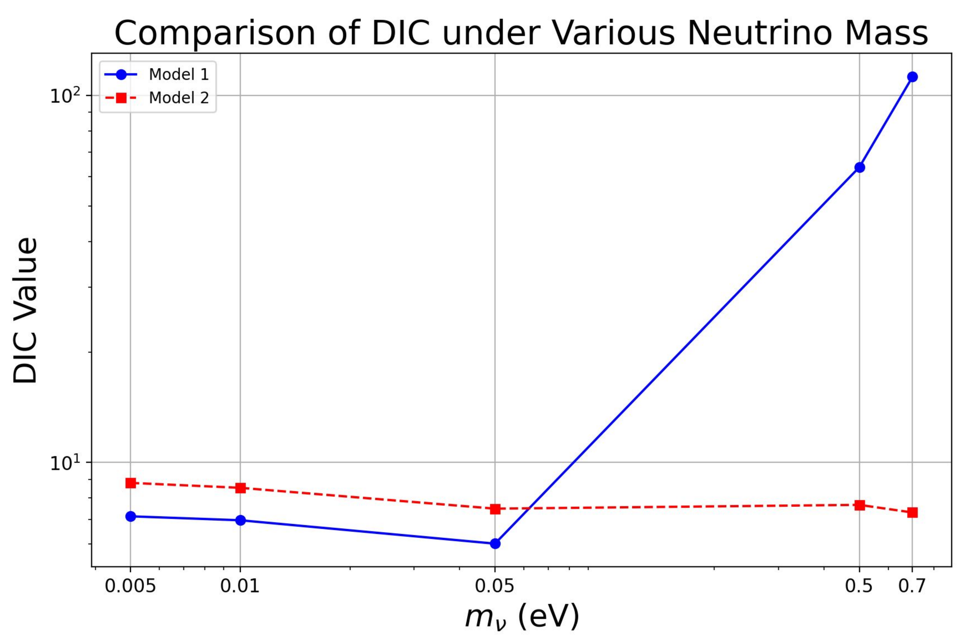
- Lower DIC indicates a better model

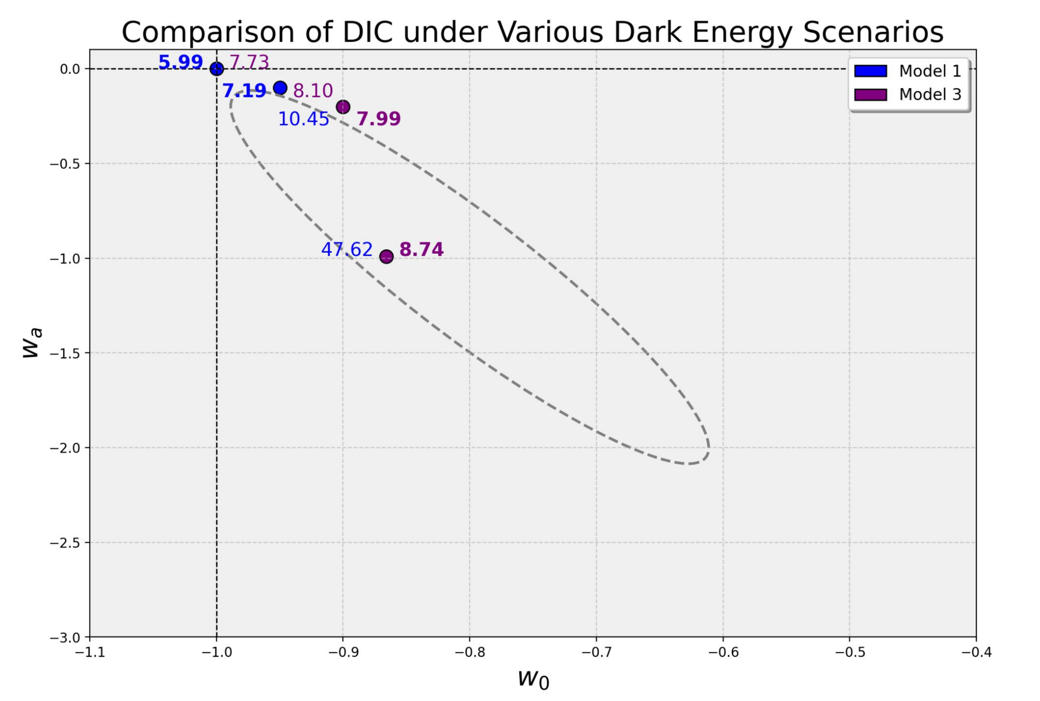
Bayes Factor:

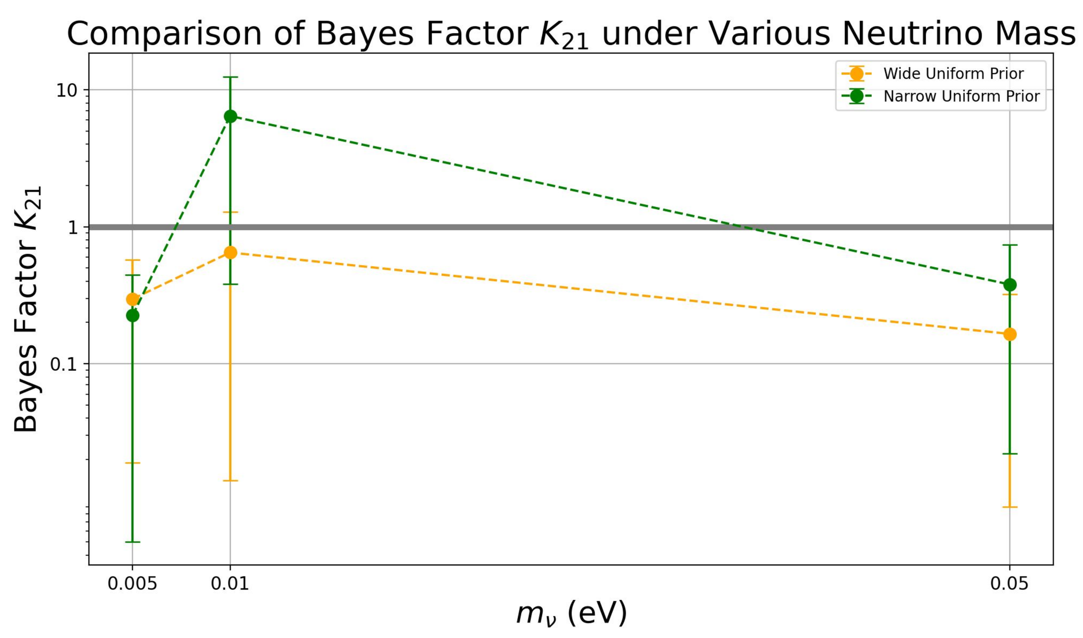
$$K_{21} = \frac{P(D \mid M_2)}{P(D \mid M_1)}$$

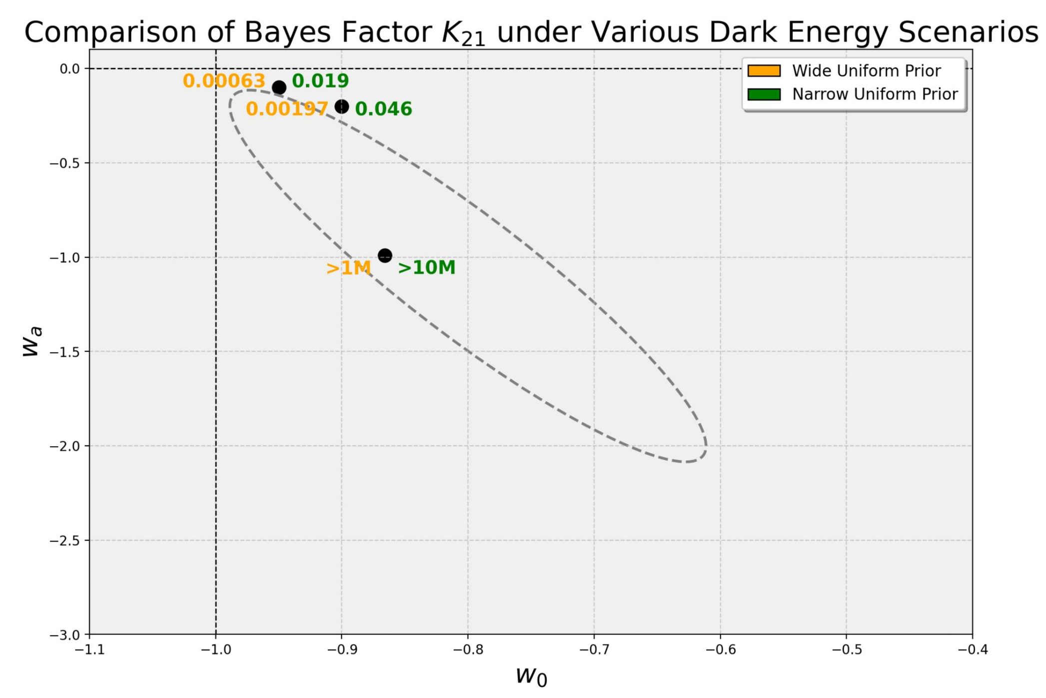
where $P(D \mid M_1)$ is the marginalized likelihood for model 1 and $P(D \mid M_2)$ is the marginalized likelihood for model 2.

- < 1 indicates stronger preference to model 1









- Wide Prior = $0 < m_\nu < 0.8$
- Narrow Prior = $0 < m_\nu < 0.4$

- Wide Prior = $-5 < w_0 < 1$; $-5 < w_a < 2$,
- Narrow Prior = $-3 < w_0 < 1$; $-3 < w_a < 2$

Conclusion

Discoveries:

- Current model comparison approaches may overlook minor fluctuations in the model
- Employing such methods without careful consideration can lead to a substantial loss of valuable information
- Proper interpretation of metrics is essential for making accurate claims during comparisons

Limitation:

- Exploring more complex models with realistic data is essential to further validate the robustness of comparison methods

Acknowledgement & References

I would like to express my gratitude to the University of Toronto Excellence Award for granting me an exceptional research opportunity.

DESI Collaboration et al. (2024). DESI 2024 VI: Cosmological Constraints from the Measurements of Baryon Acoustic Oscillations. arXiv:2404.03002.
Gelman, A., Hwang, J. & Vehtari, A. Understanding predictive information criteria for Bayesian models. Stat Comput 24, 997–1016 (2014). https://doi.org/10.1007/s11222-013-9416-2
Battersby, S. (2016). Dark energy: Staring into darkness. Nature, 537(7622), S201-S204. https://doi.org/10.1038/537S201a